mittee of the Royal Society. The first differences to be inserted in the machine can be immediately deduced from those given above; and we may hope ere long to see the logarithms of Life Tables, for single and for joint lives, printed from types cast in moulds stamped by the machine now in the course of construction by the Messrs. Donkin, for Her Majesty's Government, at the instance of the Registrar-General.

(*To be continued.*)

On the Clearing of the London Bankers. By Sir John W. Lubbock, Bart., F.R.S., *formerly Treasurer of the Royal Society, and Vice-Chancellor of the University of London.**

Atque equidem, extremo nî jam sub fine laborum
Vela traham, et terris festinem advertere proram;
Forsitan et pingues hortos quæ cura colendi
Ornaret, canerem, biferíque rosaria Pæsti:
Quóque modo potis gauderent intyba rivis;
Et virides apio ripæ, tortúsque per herbam
Cresceret in ventrem cucumis: nec sera comantem
Narcissum, aut flexi tacuissem vimen acanthi,
Pallentésque hederas, et amantes litora myrtos.

THE operation of the clearing has the effect of enabling all the payments from one bank to another to be performed without the passing of bank-notes; and the result is, that if any bank has to receive from the clearing, say, £50,000, the account of that bank at the Bank of England is better by £50,000 at 9 o'clock the next morning; and if a bank has to pay into the clearing £50,000, the amount of that bank is worse by £50,000.

The following description of the mode of conducting the clearing is taken from Mr. Babbage's *Treatise on the Economy of Machinery and Manufactures* (second edition, page 124):—

"In London all checks paid in to bankers pass through what is technically called '*The Clearing House.*' In a large room in Lombard-street, about thirty clerks from the several London bankers take their stations, in alphabetical order, at desks placed round the room; each having a small open box by his side, and the name of the firm to which he belongs in large characters on the wall above his head. From time to time, other clerks from every house enter the room, and, passing along, drop into the box the checks due by that firm to the house from which this distributor is sent. The clerk at the table enters the amount of the several checks in a book previously prepared, under the name of each bank to which it is due.

* The object of this paper is one in the attainment of which the readers of this journal are probably little interested; but the paper itself affords so remarkable an instance of the application of the doctrine of probabilities to the ordinary affairs of life, that we have not hesitated to insert it.—Ed. *A. M.*

"Four o'clock in the afternoon is the latest hour to which the boxes are open to receive checks; and at a few minutes before that time some signs of increased activity begin to appear in this previously quiet and business-like scene. Numerous clerks then arrive, anxious to distribute, up to the latest possible moment, the checks which have been paid into the houses of their employers.

"At four o'clock all the boxes are removed, and each clerk adds up the amount of the checks put into his box and due from his own to other houses. He also receives another book from his own house, containing the amounts of the checks which their distributing clerk has put into the box of every other banker. Having compared these, he writes out the balances due to or from his own house, opposite the names of each of the other banks; and having verified this by a comparison with the similar list made by the clerks of those houses, he sends to his own bank the general balance resulting from this sheet, the amount of which, if it is due from that to other houses, is sent back in bank-notes.

"At five o'clock the *Inspector* takes his seat, when each clerk, who has upon the result of all the transactions a balance to pay to various other houses, pays it to the inspector, who gives a ticket for the amount. The clerks of those houses to whom money is due, then receive the several sums from the inspector, who takes from them a ticket for the amount. Thus the whole of these payments are made by a double system of balance, a very small quantity of bank-notes passing from hand to hand, and scarcely any coin.

"It is difficult to form a satisfactory estimate of the sums which daily pass through this operation: they fluctuate from two millions to, perhaps, fifteen. About two millions and a half may possibly be considered as something like an average, requiring for its adjustment perhaps £200,000 in bank-notes and £20 in specie. By an agreement between the different bankers, all checks which have the name of any firm written across them must pass through the Clearing House; consequently, if any such check should be lost, the firm on which it is drawn would refuse to pay it at the counter; a circumstance which adds greatly to the convenience of commerce.

"The advantage of this system is such, that two meetings a day have been recently established—one at twelve, the other at three o'clock; but the payment of balances takes place once only, at five o'clock.

"If all the private banks kept accounts with the Bank of England, it would be possible to carry on the whole of these transactions with a still smaller quantity of circulating medium."

Since this was written, the system of paying in bank-notes has been completely done away with, and the amounts are settled at the Bank of England, by an entry to the debit or credit of each banker, as the case may be. There are at present 33 clearing bankers.

The details given by Mr. Babbage are not quite accurate. It is the duty of the inspector to be always present during the hours of business, to maintain order.

To calculate the probability of the concurrence of any number of independent events, the probabilities of each separately being given, is the most elementary question in that science which is called the

Theory of Probability, and is treated of in all books upon that subject.—*See* Bethune and Lubbock, *on Probability,* p. 9.* De Morgan's *Essay on Probabilities,* p. 30.

It is only necessary, in order to understand what follows, to bear in mind—

1. That the probability of the concurrence of any number of independent events is equal to the product of the probabilities of each considered separately.
2. When the number of trials is infinite, the number of times an event happens is to the number of times another independent event happens, in the ratio of their simple probabilities.

If any event in the clearing happens n times in m trials or days, I shall consider the probability of the event to be properly represented by $\frac{n}{m}$, so if I find that my house has to receive from the clearing 200 times in 400 clearings, I shall estimate the probability of that event happening on any day in future to be $\frac{200}{400}$ or $\frac{1}{2}$, and so in any other case. Strictly speaking, this probability being unknown, and only to be derived from experience, belongs to what De Morgan calls inverse probabilities; such are the probabilities which occur in questions relating to insurances on lives.

I do not pretend in these pages to give a treatise on probability, but only to show the application of the most elementary principles of that science to questions which arise in the clearing.

If a be the probability of any bank having to receive on a given day, and b the probability of that bank having to pay in, as one of these events must happen, $a+b=1$.

This is the only principle which can be predicated with certainty without recourse to observation, but it is upon the numerical values of these quantities a and b that this inquiry is based.

I think it will be admitted that $a=b$, and therefore $a=\frac{1}{2}$; at any rate, I find this to be about the case at my own bank, and I have no doubt the same obtains with all other banks, but any banker can verify it in his own case at once by reference to his pass-book.

The probability of any number of bankers having to receive on a given day is, therefore, nearly the same as the probability if thirty

* This treatise was published by the Society for the Diffusion of Useful Knowledge. When this work was written, no work on "Probability" had appeared since the time of Simpson and De Moivre. If anyone shall pretend that this work was written by De Morgan, I can produce the letter of my lamented friend with which he furnished our manuscript to Mr. Coates.

shillings are tossed that the same number of them comes heads. But as two events are equally impossible, namely, that all thirty banks either bring in or take out, these events must be excluded and the other numbers must be increased in the ratio of 1 to $1-(\frac{1}{2})^{29}$.

The next principle will, I think, also be admitted; and, at any rate, I find it in my own clearing that the probability of having to receive any given sum decreases with enormous rapidity; so that, for example, if the chance, in case of having to pay in or receive any sum between nothing and £50,000 is $\frac{1}{2}$, the chance of having to pay in or take out any sum between £50,000 and £100,000 is probably less than $\frac{1}{4}$, and so on in a rapidly decreasing ratio.

It is probable, further, that the probability decreases on each side zero in the same ratio, and that for a given bank the chance of paying in £5,000, or any given sum, is precisely the same as that of taking it out.

The next question which arises, and which is of great importance, is to obtain the accurate value of the probability of a banker paying in or taking out any particular sum. Any banker can obtain this without difficulty from his own pass-book with the Bank of England, and I think it may fairly be taken for granted that the probability for any sum within certain limits varies directly with the amount of the deposits. So that if the probability, that if a bank whose deposits are half a million takes out, it takes out a sum between 0 and £10,000, be $\frac{1}{2}$, the probability that if a bank whose deposits are a million takes out, it takes out a sum between 0 and £20,000, is also $\frac{1}{2}$, and so on. Thus this quantity, which may be called p, will vary for every bank, and as I know nothing of the phenomena which the clearing presents of any bank but my own, I am thrown upon the necessity of making the best conjectures I can from my own pass-book with the Bank of England.

The deposits of the joint-stock banks are not quite 50 millions; I conjecture the deposits of the private clearing banks to be about 50 millions. In order, therefore, to simplify the calculation, I will suppose the thirty banks to have each deposits to the amount of three millions each. I conjecture the value of p to be about $\frac{1}{2}$ for such a bank, and for any sum not exceeding £50,000.

The probability that any event will happen amongst these thirty banks is given very nearly by the corresponding term in the development of $(\frac{1}{2}+\frac{1}{2})^{30}$, so that the chance of

29 banks taking out is $30(\frac{1}{2})^{30}$ very nearly.
28 " " $\frac{30}{1}\frac{29}{2}(\frac{1}{2})^{30}$ "

Upon the supposition, therefore, that the business of each bank is so conducted that it is an even chance on any day whether any given bank has to pay or to receive in the clearing, the following table shows the chance of any given event happening on any given day, or the number of times that the event will happen in 1,000,000,000,000 days, on the average of a great many years. The upper line is given to complete the table; but as one bank at least must bring in, it is necessary to exclude the two events of no bank bringing in and no bank taking out, and the numbers in the table should be increased in the ratio of 1 to $1-(\frac{1}{2})^{29}$, that is, they should be multiplied by 1·000000002, by which, of course, they would not be sensibly altered.

No. of Bankers.	No. of Days, or Probability.		No. of Bankers.
	·0000000009313	0·9691000	
29	·0000000279397	2·4462213	29
28	·0000004051250	3·6075893	28
27	·0000037811700	4·5776260	27
26	·0000255229000	5·4069298	26
25	·0001327190000	6·1229331	25
24	·0005529960000	6·7427218	24
23	·0018959800000	7·2778350	23
22	·0054509600000	7·7364728	22
21	·0133246000000	8·1246530	21
20	·0279816000000	8·4468723	20
19	·0508756000000	8·7065096	19
18	·0805530000000	8·9060820	18
17	·1115350000000	9·0474111	17
16	·1354350000000	9·1317320	16
15	·1444640000000	9·1597609	15

The table reads thus—the extreme columns to the right and left contain the number of banks. The chance of fifteen banks having to pay in the clearing, on any given day, is ·14446. The third column gives the logarithms of the numbers in the second column.

The following table shows the number of days, in the year of 313 days, that any given event will happen, on an average:—

No. of Bankers.	No. of Days.	No. of Bankers.	No. of Days.	No. of Bankers.	No. of Days.
29		19	15·93	9	4·17
28		18	25·21	8	1·71
27		17	34·91	7	·59
26	·01	16	42·39	6	·17
25	·04	15	45·21	5	·04
24	·17	14	42·39	4	·01
23	·59	13	34·91	3	
22	1·71	12	25·21	2	
21	4·17	11	15·93	1	
20	8·76	10	8·76		

The table reads thus—in a year of 313 days, fifteen banks on an average have to pay on forty-five days, on the average of a great number of years. Of course, in order to ascertain what will take place in ten years, or 3,130 days, it is only necessary to move the decimal point one place to the right. So that in ten years seventeen banks on an average have to pay on 349 days.

I have already said, that I have little doubt that the value of a does not differ sensibly from $\frac{1}{2}$, and, if so, the values given in the table above cannot differ much from the truth. The records of the clearing would furnish the means of determining the exact value of a if there were any advantage in so doing.

The value of p is subject to greater difficulty; it may be different for different banks, and so may the value of a; but the conditions of the problem are such as to render it evident that if a is less than $\frac{1}{2}$ for one bank it is probably greater than $\frac{1}{2}$ for another, and unless its average value for all the thirty banks differs widely from $\frac{1}{2}$, my results will not be affected. The same remark applies also to my value of p; it may, and no doubt it does, differ very much for every bank, varying with their mode of conducting the business, and with the magnitude of their deposits. But it is the average value with which we are mainly concerned, and unless its value differs considerably on the average of all the banks from the value which I shall assign to it, my results will still hold good.

One thing I hold to be certain with regard to p, which is, that its value decreases rapidly; so that, if its value for any given bank and for any given b pounds is p, the value for the b following pounds is far less than p.

The greatest difficulty we have to contend with in determining the value of p, arises from the possibility of its varying with the state of the times, that is, with the state of the money-market; and it is, probably, some unknown function of the rate of discount. I do not think it can vary with the rate of discount; it may, perhaps, be higher in times of panic than at others, but I do not think so. It may, of course, be very much higher for days when the dividends are being paid, and when the clearing is affected by any large operation. It would be difficult to determine the value of p for any sum within small limits, and for any given bank, say, for £1,000, but, of course, the wider the limits the less danger of error.

I think, for a bank holding £3,000,000 deposits, and for a sum not exceeding £50,000, the value of p does not differ materially from $\frac{1}{2}$. Of course, if I knew the returns of the clearing-house I could tell its value, and for any given bank I could easily ascertain

its value from their bank pass-book, but with only my own experience to guide me in this difficult determination, I give this value with much diffidence. I wish the reader to bear in mind, that while I am groping my way in the dark, a few minutes' examination of the clearing-house returns, properly tabulated, would give a clear insight into this and all similar questions.

Upon this hypothesis, the chances of the Bank having to pay more than £50,000 is, of course, also $\frac{1}{2}$, and the chance of fifteen banks taking out each £50,000 is the probability given in the following table, multiplied by $(\frac{1}{2})^{15}$. So that the probabilities given in the table are enormously reduced, and the probability of fifteen banks having to pay, which is

$$\cdot 14446$$

an event which would happen 14,446 times in 100,000 trials, upon the supposition that, if all the fifteen banks have to pay at all, they have to pay at least £50,000, becomes very much reduced.

Mr. Farley has kindly examined the pass-book of my bank with the Bank of England, and, from the results Mr. Farley obtained, I conjecture the following numbers to apply to a bank whose deposits are about £3,000,000:—

	Probability.	Logarithm of Probability.	Probability.	Logarithm of Probability.	
£					£
250,000	·01383	8·1408155	·01383	8·1408155	250,000
200,000	·01330	8·1237822	·02713	8·4334498	200,000
150,000	·03298	8·5182338	·06011	8·7789467	150,000
100,000	·07074	8·8496938	·13085	9·1167737	100,000
50,000	·14841	9·1714464	·27926	9·4460037	50,000
0,000	·22074	9·3438903	..	..	0,000
0,000	·22074	9·3438903	..	..	0,000
50,000	·14841	9·1714464	·27926	9·4460037	50,000
100,000	·07074	8·8496938	·13085	9·1167737	100,000
150,000	·03298	8·5182338	·06011	8·7789467	150,000
200,000	·01330	8·1237822	·02713	8·4334498	200,000
250,000	·01383	8·1408155	·01383	8·1408155	250,000

The first column gives the sum in thousands.

The second column gives the number of times the event happens in 100,000 trials.

The third column gives the logarithm of the number in the second column.

The table reads thus:—A bank with £3,000,000 deposits has to receive between £50,000 and £100,000 14,841 times in 100,000 trials or clearings. The chance of a bank holding £3,000,000

deposits having to pay above £50,000 is ·27926, the logarithm of which is 9·4460037.

I apprehend this law or curvature of p will be similar for all banks, or nearly so; but, of course, for banks whose operations are greater, their deposits being also greater, the figures in the first column will be proportionably greater.

The numbers in the third column are nearly in a geometrical ratio, of which the common ratio is $\frac{1}{2}$, confirming the opinion given in p. 146, of the rapidity with which these numbers decrease on each side zero.

It is evident, that the chances of large sums being taken out of the clearing diminish, for two reasons: the one is, the great improbability of the number of bankers who have to receive greatly exceeding the number of those who have to pay; and secondly, that for each bank the chance of having a moderate sum to pay greatly exceeds that of having a large sum to pay.

Supposing the values of the probability to be approximately correct, it is easy to find the chance of the clearing amounting to any given sum on a given day.

At present the London bankers are compelled to keep very large sums of money unemployed, in order to provide for the possible results of each day's operations. It is, of course, impossible to ascertain the amount of notes kept in the tills of the several banks, but the returns presented to Parliament give, for each week, the total amounts of the balances kept by the bankers with the Bank of England, up to the end of 1857.

A large amount might, therefore, safely be employed by the bankers collectively, and a considerable profit obtained, if the clearing could be worked out of a common fund, so as to assimilate the position of the banks to what it would be if they were all united in one establishment.

In 1839, the actual transfer of money, or *difference of the sides*, was not, upon the average, more than £20,000, and now, probably, £500,000 would be sufficient to provide for this payment. Although, however, the average difference is so small, the uncertainty is so great, that the London bankers are compelled to keep at the Bank of England a balance which varies between £2,500,000 and £4,000,000, and, probably, never falls below the smaller amount.

It is somewhat remarkable, that during the panic at the end of 1857 the balances of the London bankers exceeded £6,000,000.

It is probable also, that independent of the actual profit, the

collateral advantages of a combination of the bankers, and the working the payments in the clearing out of a joint account, instead of thirty-three separate accounts at the Bank of England, would be by no means inconsiderable. Although the London bankers enjoy the confidence of the public, still they would, if united, be able more effectually to provide against any panic, and to prevent the recurrence of a state of events such as that which in November, 1857, endangered the whole of our monetary system.

As my career is drawing to a close, or, in the words of the poet, as my bark is nearing the shore, I must leave the details of such a plan to be worked out by younger heads.

On some Considerations suggested by the Annual Reports of the Registrar-General, being an Inquiry into the Question as to how far the Inordinate Mortality in this Country, exhibited by those Reports, is controllable by Human Agency. (Part II.) By H. W. PORTER, ESQ., B.A., *Assistant Actuary to the Alliance Assurance Company, Fellow of the Institute of Actuaries and of the Statistical Society.*

(Concluded from page 112.)

AS regards the hereditary transmission of phthisis, another investigation of the statistics of the Hospital for Consumption at Brompton showed, that where one parent only was affected with pulmonary disease, the fathers being so affected transmitted the disease to their sons in 63 out of 106 cases, being 59·4 per cent. of the whole number observed; and to their daughters in 47 cases only out of 108, being 43½ per cent.: while the mothers being phthisical transmitted the disease to their sons in 43 cases, being 40·6 per cent.; and to their daughters in 61 cases, being 56½ per cent. of the cases under observation. Judging, therefore, from these figures, it would probably appear, if a large number of cases were registered, that the power of transmission of disease by phthisical fathers to their sons, and by phthisical mothers to their daughters, is about the same.

Very similar—in fact, almost identical—results are shown by a similar comparison of the statistics of insanity.

From the consideration of such facts as these, it appears that we hold in our hands the power of checking the increase of diseases of an hereditary character by discouraging marriages under certain circumstances—the extreme inexpediency of which, in some cases, seems clearly apparent; and Life Assurance Companies, by